Losses on Multiconductor Transmission Lines in Multilayered Dielectric Media

ROGER F. HARRINGTON, FELLOW, IEEE, AND CAO WEI

Abstract —For the transmission-line modes, a multiconductor transmission line in a multilayered dielectric medium can be characterized by four matrix parameters: the capacitance matrix $[C]$, the inductance matrix $[L]$, the shunt conductance matrix $[G]$, and the series resistance matrix $[R]$. The first two matrices $[C]$ and $[L]$ can be obtained from equivalent electrostatic and magnetostatic problems. The conductance matrix $[G]$ can be obtained by changing all dielectric constants ϵ_i to complex dielectric constants $\hat{\epsilon}_i$ in the equivalent electrostatic problem. The resistance matrix $[R]$ can be obtained by applying a perturbation method to each mode of the transmission line. A computer program has been written for an arbitrary line, and sample computations are presented.

I. Introduction

To a good approximation, the transmission-line properties of a multiconductor transmission line in a multilayered dielectric medium can be characterized by four matrix parameters: the capacitance matrix $[C]$, the inductance matrix $[L]$, the shunt conductance matrix $[G]$, and the series resistance matrix $[R]$. The first matrix $[C]$ can be obtained by solving an electrostatic problem involving electric potential and electric charge. The second matrix $[L]$ can be obtained by solving a magnetostatic problem involving magnetic flux linkage and electric current. This magnetostatic problem has an electrostatic analogue which can be solved as an electrostatic problem with ϵ replaced by μ. The theory for calculating $[C]$ and $[L]$, and sample computations, are given in [1]. The computer programs used for the computations are given in [2].

The solution for the shunt conductance matrix $[G]$ can be obtained in a simple manner by considering the dielectric constant ϵ_i of the various dielectrics to be complex. This results in a complex matrix $[\hat{C}]$, the real part of which is $[C]$ and the imaginary part of which is related to $[G]$ according to $[G]=\text{Re}[j\omega\hat{C}]$.

The solution for the series resistance matrix is more complicated. On a transmission line with n conductors plus ground there can exist n modes of propagation. Each mode, which normally consists of voltages and currents on all wires, propagates exponentially along the line. The usual perturbation analysis for conductor losses that applies to single-mode lines [3], [4] can be applied to each mode of the multiconductor transmission line for the determination of attenuation constants. Once the attenuation constants are found, the $[R]$ matrix of the multiconductor line can be obtained by solving a set of simultaneous equations.

The four matrixes $[C]$, $[L]$, $[R]$, and $[G]$ completely characterize the transmission-line behavior (to the TEM approximation) in the frequency domain according to multiconductor transmission-line theory [4], [5]. The situation is not so simple in the time domain. The lossy line no longer has the time-delayed wave solutions that it had in the loss-free case. Two possible methods for obtaining time domain solutions for the lossy line are: a) Fourier transformation to the frequency domain, solution in the frequency domain, and inverse Fourier transformation to the time domain; and b) discretation of line length variable and time, reduction of the equations to state-space form, and marching on in time.

II. Basic Theory

The transmission-line systems we wish to consider are either N conductors in an M-layered dielectric medium above a ground plane, as shown in Fig. 1, or N conductors in an M-layered dielectric medium between two ground planes, as shown in Fig. 2. The conductors can be of arbitrary shape and of either finite thickness or zero thickness. The basic formulation for the capacitance matrix $[C]$ and the inductance matrix $[L]$ for the lines is given in [1] and [2]. In the present paper we take these solutions and extend them to calculate the shunt conductance matrix $[G]$ and the series resistance matrix $[R]$.

The basic transmission-line equations for a multiconductor transmission line in the frequency domain ($e^{j\omega t}$ time variation) are

$$-\frac{\partial \vec{I}}{\partial z}=[G+j\omega C]\vec{V} \tag{1}$$

$$-\frac{\partial \vec{V}}{\partial z}=[R+j\omega L]\vec{I}. \tag{2}$$

Here $\vec{I}$ is the column vector of line currents, $\vec{V}$ is the column vector of line voltages, z is the axis of the line, and ω is the radian frequency. The first equation is a statement of the conservation of charge, and the second equation is a statement of Faraday's law of induction.

The complex power in the $+z$ direction is given by

$$P=\tilde{I}^*\vec{V} \tag{3}$$

Manuscript received November 10, 1983; revised February 21, 1984. This work was supported by the Digital Equipment Corporation, Marlboro, MA 01752.

R. F. Harrington is with the Department of Electrical and Computer Engineering, Syracuse University, Syracuse, NY 13210.

C. Wei is with Syracuse University, Syracuse, NY 13210, on leave of absence from the Nanjing Institute of Posts and Telecommunications, Nanjing, China.

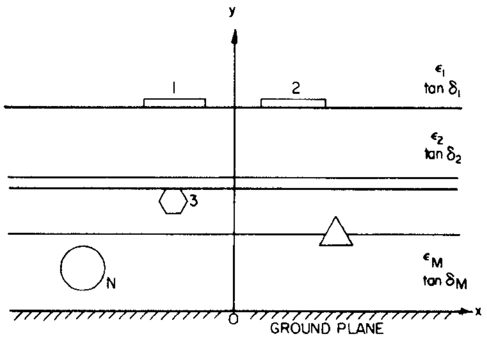

Fig. 1. A multiconductor transmission line in a multilayered dielectric medium above a ground plane.

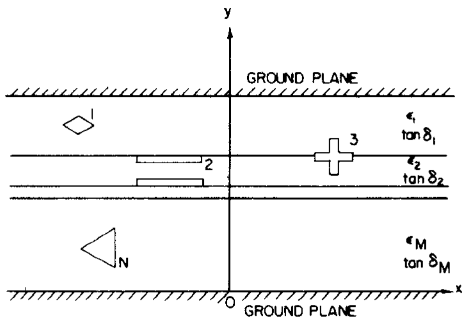

Fig. 2. A multiconductor transmission line in a multilayered dielectric medium between two ground planes.

where $\tilde{I}^*$ is the transpose (~) conjugate (*) of $\vec{I}$, and rms values of current and voltage are used. The real part of P is the time-average power transmitted by the line

$$P_T = \mathrm{Re}(\tilde{I}^*\vec{V}). \tag{4}$$

The rate of decrease in P with z is given by

$$-\frac{\partial P}{\partial z} = -\frac{\partial}{\partial z}(\tilde{I}^*\vec{V}) = -\tilde{I}^*\frac{\partial \vec{V}}{\partial z} - \frac{\partial \tilde{I}^*}{\partial z}\vec{V}. \tag{5}$$

Substituting from (1) and (2), we obtain

$$-\frac{\partial P}{\partial z} = \tilde{I}^*[R + j\omega L]\vec{I} + \tilde{V}^*[\tilde{G} + j\omega\tilde{C}]^*\vec{V}. \tag{6}$$

The transposes on $[\tilde{G}]$ and $[\tilde{C}]$ can be dropped since they are symmetric matrices. The real part of $-\partial P/\partial z$ is the time-average power lost per unit length of line, that is:

$$P_L = \mathrm{Re}\left(-\frac{\partial P}{\partial z}\right) = P_C + P_D = \tilde{I}^*[R]\vec{I} + \tilde{V}^*[G]\vec{V}. \tag{7}$$

For any single mode, the ratio P_L/P_T is twice the attenuation constant of that mode, the same as for the single-mode line [4], [5].

To obtain the modes of the transmission line, we seek solutions which vary as

$$\vec{I} = \hat{I}e^{j\omega t - \gamma z} \tag{8}$$

$$\vec{V} = \hat{V}e^{j\omega t - \gamma z} \tag{9}$$

where γ is the propagation constant. Substituting these into (1) and (2), we have

$$\gamma\hat{I} = [G + j\omega C]\hat{V} \tag{10}$$

$$\gamma\hat{V} = [R + j\omega L]\hat{I}. \tag{11}$$

Substituting for $\hat{V}$ from (11) into (10), and vice versa, we obtain

$$[G + j\omega C][R + j\omega L]\hat{I} = \gamma^2\hat{I} \tag{12}$$

$$[R + j\omega L][G + j\omega C]\hat{V} = \gamma^2\hat{V}. \tag{13}$$

These are eigenvalue equations for γ^2. Since $[C]$, $[L]$, $[G]$, and $[R]$ are all symmetric matrices, the left-hand coefficient matrix of (12) is the adjoint (with respect to the inner product) of the left-hand coefficient matrix of (13), and vice versa. Hence, the eigenvalues γ^2 obtained from (12) must be equal to those obtained from (13). The eigenvectors $\hat{I}$ obtained from (12) will not be equal to the eigenvectors $\hat{V}$ obtained from (13). Rather $\hat{I}$ will be the right-hand eigenvectors of the unsymmetric matrix $[G + j\omega C][R + j\omega L]$ and $\hat{V}$ will be the left-hand eigenvectors. The eigenvectors $\hat{I}$ and $\hat{V}$ cannot be chosen independently of each other, but instead must be chosen to satisfy (10) and (11).

III. The Conductance Matrix

The simplest way to evaluate the conductance matrix $[G]$ is to replace the real dielectric constants ϵ_i in the loss-free solution [1], [2] by complex dielectric constants [6]

$$\hat{\epsilon}_i = \epsilon_i' - j\epsilon_i'' = \epsilon_i'(1 - j\tan\delta_i). \tag{14}$$

Here, ϵ_i' and $-\epsilon_i''$ are the real and imaginary parts of $\hat{\epsilon}_i$, and $\tan\delta_i = \epsilon_i''/\epsilon_i'$ is the loss tangent. The imaginary part of $\hat{\epsilon}$ is sometimes written in terms of a conductivity σ as $\epsilon'' = \sigma/\omega$, but this is somewhat misleading since the losses are usually not due to conduction current.

Using the complex dielectric constants in the previous solution [1], [2], we obtain a complex "capacitance" matrix $[\hat{C}]$ for the transmission-line system. This is actually the complex matrix appearing in (1), or

$$j\omega[\hat{C}] = [G + j\omega C]. \tag{15}$$

The conductance is just the real part of (15), or

$$[G] = \mathrm{Re}[j\omega\hat{C}] = -\mathrm{Im}(\omega\hat{C}). \tag{16}$$

The capacitance matrix for the lossy line is

$$[C] = \mathrm{Re}[\hat{C}]. \tag{17}$$

For lines with low dielectric losses, the $[C]$ of (17) should be approximately equal to that for the loss-free case. To summarize, we calculate $[G + j\omega C]$ for the lossy line by using the solution previously developed [1], [2], allowing all dielectric constants to be complex, obtaining the complex $[\hat{C}]$, and finally applying (15).

It is often more important to obtain the attenuation constants of the modes than the $[G]$ matrix itself. To do this, we use the relationship

$$\alpha_D = \frac{P_D}{2P_T} \tag{18}$$

where P_T is given by (4) and P_D by

$$P_D = \tilde{V}^*[G]\vec{V}. \quad (19)$$

It should be emphasized that both P_T and P_D must be calculated for each particular mode. If the dielectric losses are small, we can use the loss-free modes to calculate both P_T and P_D, similar to the procedure used in the next section.

IV. The Resistance Matrix

The usual way to compute the conductor losses on a single-mode line is to calculate the attenuation constant by a perturbation approach [5]. To extend this solution to a multiconductor transmission line, we should apply this approach separately to each mode. Once we have all the modal attenuation constants, we can calculate the resistance matrix for the line.

We start with the modes of the loss-free line, which are solutions to the eigenvalue equations

$$[C][L]\hat{I}^o = \frac{1}{v_p^2}\hat{I}^o \quad (20)$$

$$[L][C]\hat{V}^o = \frac{1}{v_p^2}\hat{V}^o \quad (21)$$

where the superscripts o denote unperturbed, and

$$v_p = \frac{\omega}{\beta} \quad (22)$$

is the phase velocity, with β the phase constant ($\gamma = j\beta$). Equations (20) and (21) are (12) and (13) with $[R]=[G]=0$. Again, $\hat{I}^o$ and $\hat{V}^o$ must be related by (10) and (11), which become

$$\hat{I}^o = v_p[C]\hat{V}^o \quad (23)$$

$$\hat{V}^0 = v_p[L]\hat{I}^o \quad (24)$$

in the loss-free case.

It is important to note that the phase velocities v_p are generally different for each mode, since the transmission line has several different dielectrics. Such a line is referred to by Kajfez [3] as a multivelocity transmission line. All we know for sure is that the v_p satisfy $c \geqslant v_p \geqslant c/\sqrt{\epsilon_{max}}$, where c is the velocity of light and ϵ_{max} is the largest dielectric constant present.

Once we solve (20)–(24) for the unperturbed mode voltages and currents, we can calculate the unperturbed power flow from (4) as

$$P_T^o = \tilde{I}^o\vec{V}^o. \quad (25)$$

Note that the Re operator and the conjugate on $\tilde{I}$ can be dropped since the unperturbed $\hat{V}^o$ and $\hat{I}^o$ are nearly real. The power loss per unit length of the transmission line can be calculated by the usual perturbation formula

$$P_C \approx \sum_k R_S \int_{l_k} J_k^2\, dl \quad (26)$$

where R_S is the surface resistance of the metal, J_k is the current density on the kth conductor, and the integral is taken over all metal surfaces l_k. Equation (26) is evaluated by using the moment solution [1], [2] in a summation approximation to the integral. Once P_C is evaluated for a given mode, the attenuation due to conductor losses is approximately given by

$$\alpha_C \approx \frac{P_C}{2P_T^o}. \quad (27)$$

Again (27) is valid only for each mode, not for arbitrary excitations.

Finally, once we have the attenuation constants for each mode, we can evaluate the resistance matrix as follows. From (13) with $[G]=0$ we have

$$(\alpha + j\beta)^2\hat{V} = [R + j\omega L][j\omega C]\hat{V}. \quad (28)$$

If the losses are low, we can take $\hat{V} \approx \hat{V}^o$ to be real. The imaginary part of (28) is then

$$2\alpha\beta\hat{V}^o = \omega[R][C]\hat{V}^o. \quad (29)$$

Finally, setting $\beta \approx \beta^o$ and using (23), this reduces to

$$2\alpha\hat{V}^o \approx [R]\hat{I}^o. \quad (30)$$

Since we now know α, $\hat{V}^o$, and $\hat{I}^o$ for each mode, (30) is sufficient for calculating $[R]$. To be specific, for mode i let $\alpha, \hat{V}^o, \hat{I}^o$ be denoted $\alpha_C^i, \hat{V}^i, \hat{I}^i$, and write (30) as

$$\begin{aligned} 2\alpha_C^i\hat{V}_1^i &= R_{11}\hat{I}_1^i + R_{12}\hat{I}_1^i + \cdots + R_{1N}\hat{I}_N^i \\ 2\alpha_C^i\hat{V}_2^i &= R_{21}\hat{I}_1^i + R_{22}\hat{I}_2^i + \cdots + R_{2N}\hat{I}_N^i \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ 2\alpha_C^i\hat{V}_N^i &= R_{N1}\hat{I}_1^i + R_{N2}\hat{I}_2^i + \cdots + R_{NN}\hat{I}_N^i \\ i &= 1,2,\cdots,N. \end{aligned} \quad (31)$$

If we take the jth equation from (30) for each i, we have

$$\begin{aligned} R_{j1}\hat{I}_1^1 + R_{j2}\hat{I}_2^1 + \cdots + R_{jN}\hat{I}_N^1 &= 2\alpha_C^1\hat{V}_j^1 \\ R_{j1}\hat{I}_1^2 + R_{j2}\hat{I}_2^2 + \cdots + R_{jN}\hat{I}_N^2 &= 2\alpha_C^2\hat{V}_j^2 \\ \cdots\cdots\cdots\cdots\cdots\cdots& \\ R_{j1}\hat{I}_1^N + R_{j2}\hat{I}_2^N + \cdots + R_{jN}\hat{I}_N^N &= 2\alpha_C^n\hat{V}_j^n \\ j = 1,2,\cdots,N.& \end{aligned} \quad (32)$$

These are N^2 equations for the N^2 unknowns $[R]$. Hence, once the α_C^i are determined, we can calculate all R_{ij} for the multiconductor transmission line.

As a word of caution, note that $\hat{V}$ and $\hat{I}$ cannot both be taken as real in (10) or (11) to calculate α. If this were done, the resulting left-hand side of (30) would be wrong. (The factor of 2 would be missing.) The reason for this error lies in the fact that $\hat{V}$ and $\hat{I}$ are slightly out of phase, by an amount just sufficient to account for the factor of 2.

V. Losses on the Ground Planes

In the moment solution [1], [2], the current is determined on the conducting lines and on the upper ground plane, if present, but not on the lower ground plane. This is because image theory is used to account for the current on the lower ground plane. Hence, to include in (26) the loss on the lower ground plane, we must first determine the current on it.

The magnetic field $\boldsymbol{H}$ from an infinitely long filament of current is given by the Biot–Savart law. If we consider the

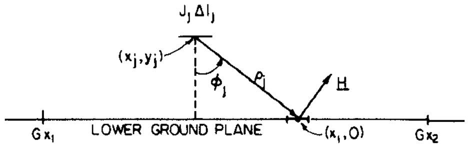

Fig. 3. Coordinate system for a subsection of current above the lower ground plane.

subsection of current $J_j \Delta l_j$ in free space to be a filament, its magnetic field has only a ϕ_j component given by

$$H_{\phi_j} = \frac{J_j \Delta l_j}{2\pi \rho_j} \tag{33}$$

where (ρ_j, ϕ_j) are the local polar coordinates (see Fig. 3). The tangential component of $\boldsymbol{H}$ at the ground plane is

$$H_t = H_{\phi_j} \cos \phi_j = \frac{J_j \Delta l_j}{2\pi \rho_j}\left(\frac{y_j}{\rho_j}\right) \tag{34}$$

where (x_j, y_j) are the local rectangular coordinates of $J_j \Delta l_j$ (see Fig. 3). The tangential component of H at a point $(x_i, 0)$ due to all elements of current (including those on the upper ground plane, if present) above the lower ground plane is the sum of (34) over all elements, or

$$H_t(x_i, 0) = \frac{1}{2\pi} \sum_j \frac{J_j \Delta l_j y_j}{(x_i - x_j)^2 + y_j^2}. \tag{35}$$

The H_t due to all image currents is also equal to (35), hence the total H_t at $(x_i, 0)$ is just twice that of (35). Hence, the current J_{iLG} at a point $(x_i, 0)$ on the lower ground plane is

$$J_{iLG} = \frac{1}{\pi} \sum_j \frac{J_j \Delta l_j y_j}{(x_i - x_j)^2 + y_j^2}. \tag{36}$$

We use this result to numerically evaluate the losses in the lower ground plane.

If the ground planes are perfect, we include in (26) only that current on the transmission lines. If there exists an imperfect lower ground plane, we include the current elements $J_{iLG} \Delta l_{iLG}$ in (26) in addition to the current on the lines. If there also exists an imperfect upper ground plane, its losses are included in (26) in the same manner as the losses on the lines. For the evaluation of losses on the lower ground plane, we include a width of it equal to 2 or 3 times the transverse extent of the transmission lines.

The incremental inductance rule, together with numerical differentiation could be used as suggested by Wheeler [7]. However, the numerical integration used in this paper is easier to implement in a general computer program.

VI. Numerical Examples

A general computer program has been written for multiconductor transmission lines in multilayered dielectric media. Some examples are given in this section to test the program. We compare our results to those available in the literature whenever possible.

Example 1

Consider a single conducting line of circular cross section, diameter $d = 1$, and a distance H above a perfect

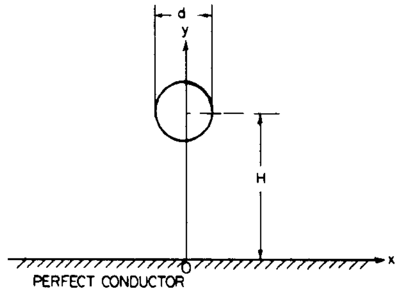

Fig. 4. A circular conductor above a perfectly conducting ground plane.

TABLE I
Comparison of (A) the Numerical Solution to (B) the Analytical Solution for the Transmission Line of Fig. 4

H	Solution	C(F/m.)	R(/m.)	G(℧/m.)
2	(A)	1.073×10^{-10}	0.8611×10^{-3}	0.8091×10^{-4}
	(B)	1.078×10^{-10}	0.8580×10^{-3}	0.8131×10^{-4}
3	(A)	0.8944×10^{-10}	0.8459×10^{-3}	0.6744×10^{-4}
	(B)	0.8980×10^{-10}	0.8426×10^{-3}	0.6771×10^{-4}
4	(A)	0.8008×10^{-10}	0.8407×10^{-3}	0.6038×10^{-4}
	(B)	0.8037×10^{-10}	0.8374×10^{-3}	0.6060×10^{-4}
5	(A)	0.7410×10^{-10}	0.8385×10^{-3}	0.5587×10^{-4}
	(B)	0.7434×10^{-10}	0.8350×10^{-3}	0.5605×10^{-4}

ground plane as shown in Fig. 4. The dielectric medium surrounding the line is infinite in extent with dielectric constant $\epsilon_r = 4$ and loss tangent $\tan\delta = 1.2 \times 10^{-3}$. The circular conductor is made of copper for which the surface resistance is $R_s = 2.61 \times 10^{-7}\sqrt{f}$, and the frequency f is taken to be 100 MHz. The numbers of subsections chosen are 20 for the circular conductor and 30 for the ground plane from $x = -1.5H$ to $x = 1.5H$, where H is the distance from the ground plane to the axis of the circular conductor.

Analytical formulas for the L, C, R, and G parameters of the two-wire line can be found in [5, table 9.01]. For the single-wire line over ground, L and R are one-half those values, and C and G are twice those values, or

$$L = \frac{\mu}{2\pi} \cosh^{-1}(2H/d) \tag{37}$$

$$C = \frac{2\pi\epsilon}{\cosh^{-1}(2H/d)} \tag{38}$$

$$R = \frac{R_s}{\pi d}\left[\frac{2H/d}{\sqrt{(2H/d)^2 - 1}}\right] \tag{39}$$

$$G = \frac{2\pi\omega\epsilon \tan\delta}{\cosh^{-1}(2H/d)}. \tag{40}$$

A comparison of the results computed from our program with the above analytical solution is given in Table I. The value of C computed from the complex dielectric constant agrees with that computed from a real dielectric constant

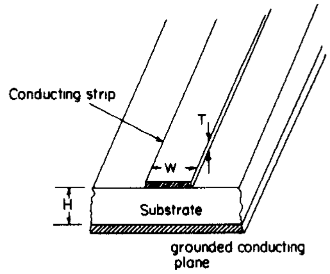

Fig. 5. A microstrip transmission line.

TABLE II
RESULTS COMPUTED WITH THE GENERAL PROGRAM FOR THE MICROSTRIP OF FIG. 5

W/H	0.1	0.2	0.3	0.4	0.6	1.0	1.2	1.4	2.0
R(Ω/m)	0.06605	0.03993	0.02986	0.02400	0.01848	0.01315	0.01173	0.01063	0.00843
$\frac{\alpha_c Z_o H}{R_s}$(dB)	21.981	13.289	9.937	8.120	6.150	4.376	3.904	3.538	2.807

using the previous program [1] to more than four significant figures in each case.

Example 2

Next consider the microstip shown in Fig. 5. The stripline and the ground plane are both made of copper, with surface resistance $R_s = 2.61 \times 10^{-7}\sqrt{f}$. The frequency f is taken to be 1 MHz. The substrate has a dielectric constant $\epsilon_r = 11.7$ and a loss tangent $\tan\delta = 10^{-3}$. Above the strip, $\epsilon_r = 1$ and $\tan\delta = 8 \times 10^{-4}$. The thickness of the substrate is $H = 0.02$ and the thickness of strip is $T = 0.02H$.

For the program, we use 26 subsections for the strip contour, 12 subsections from $x = -1.5$ $[W, H]$ to $x = -0.5W$ on the dielectric interface to the left of the strip, and 12 subsections from $x = 0.5W$ to $x = 1.5$ $[W, H]$ on the dielectric interface to the right of the strip. Here $[W, H]$ denotes the larger of W and H. The numerical results are listed in Table II. The last row of the table is based on the relationship

$$\alpha_C = 4.343 \frac{R}{Z_o} \quad \text{(dB/m)} \tag{41}$$

where Z_o is the characteristic impedance of the stripline. Fig. 6 shows a plot of the last column of Table II compared with the same parameter obtained in [8] by an analytical approach. The agreement between these two very different methods of solution is good.

Example 3

To illustrate the generality of the solution and computer program, the example shown in Fig. 7 is considered. The numbers of subsections used are 8 for the circular conductor, 6 for the other three conductors, 15 for the upper ground plane from $x = -1.5$ to $x = 1.5$, 8 for the upper left dielectric interface from $x = -1.5$ to $x = 0.1$, 6 for the upper right dielectric interface from $x = 0.3$ to $x = 1.5$, 5 for both the lower left and right dielectric interfaces from $x = -1.5$ to $x = -0.5$ and from $x = 0.5$ to $x = 1.5$, and 2 for the lower middle dielectric interface from $x = -0.1$ to $x = 0.1$. The conducting material is copper for which $R_s = 2.61 \times 10^{-7}\sqrt{f}$, and the frequency is 1 MHz. The computed results are listed on the following two pages of computer output. No alternative solution can be given for comparison. The upper rectangular conductor is #1, the triangular conductor #2, the lower rectangular conductor #3, and the circular conductor #4.

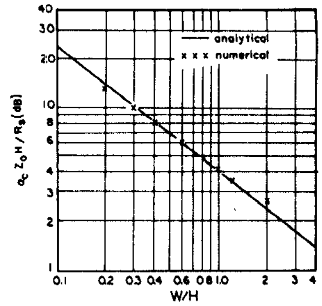

Fig. 6. Comparison of numerical and analytical [8] solutions for conductor attenuation on a microstrip transmission line ($T/H = 0.02$).

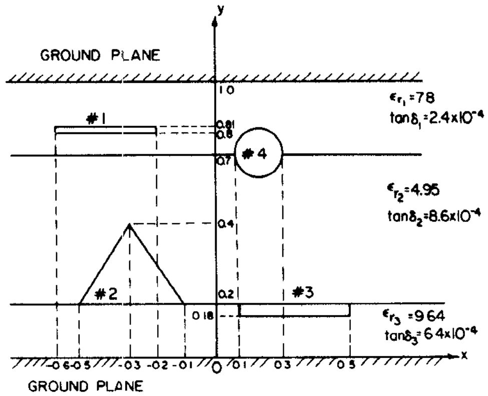

Fig. 7. Four conducting transmission lines in a three-layered dielectric medium between two ground planes.

TABLE III
THE MATRICES $[C]$, $[L]$, $[G]$, AND $[R]$ FOR THE MULTICONDUCTOR TRANSMISSION LINE OF FIG. 7

I	J	C(I,J)	L(I,J)	G(I,J)	R(I,J)
1	1	0.3088E-09	0.2714E-06	0.6420E-06	0.7375E-03
1	2	-0.3606E-10	0.4488E-07	-0.1701E-06	0.8518E-04
1	3	-0.2848E-11	0.1295E-07	-0.1327E-07	0.4403E-04
1	4	-0.2459E-10	0.3442E-07	-0.6070E-07	0.6033E-04
2	1	-0.3606E-10	0.4488E-07	-0.1701E-06	0.8518E-04
2	2	0.3331E-09	0.2594E-06	0.1481E-05	0.6801E-03
2	3	-0.3045E-10	0.3447E-07	-0.1498E-06	0.8758E-04
2	4	-0.1641E-10	0.3742E-07	-0.9688E-07	0.1401E-03
3	1	-0.2848E-11	0.1295E-07	-0.1327E-07	0.4403E-04
3	2	-0.3045E-10	0.3447E-07	-0.1498E-06	0.8758E-04
3	3	0.3806E-09	0.2570E-06	0.1608E-05	0.6711E-03
3	4	-0.3178E-10	0.5248E-07	-0.1777E-06	0.1604E-03
4	1	-0.2459E-10	0.3442E-07	-0.6070E-07	0.6033E-04
4	2	-0.1641E-10	0.3742E-07	-0.9688E-07	0.1401E-03
4	3	-0.3178E-10	0.5248E-07	-0.1777E-06	0.1604E-03
4	4	0.2328E-09	0.3326E-06	0.6383E-06	0.6901E-03

TABLE IV
THE EIGENVALUES, ATTENUATION CONSTANTS, AND EIGENVECTORS FOR THE MULTICONDUCTOR TRANSMISSION LINE OF FIG. 7

MODE NUMBER 1	EIGEN. = 0.8775E+01	ALPHA = 0.1296E-04
MODE NUMBER 2	EIGEN. = 0.7675E+01	ALPHA = 0.1174E-04
MODE NUMBER 3	EIGEN. = 0.7061E+01	ALPHA = 0.1277E-04
MODE NUMBER 4	EIGEN. = 0.6529E+01	ALPHA = 0.8297E-05

Mode voltages	Mode currents
V(1,1) = 0.1794E+00	I(1,1) = 0.3239E-02
V(2,1) = 0.3319E+00	I(2,1) = 0.7304E-02
V(3,1) = 0.8506E+00	I(3,1) = 0.3051E-01
V(4,1) = 0.3662E+00	I(4,1) = 0.4892E-02
V(1,2) = 0.6429E+00	I(1,2) = 0.1845E-01
V(2,2) = 0.7087E+00	I(2,2) = 0.2365E-01
V(3,2) = -0.2590E+00	I(3,2) = -0.1365E-01
V(4,2) = 0.1318E+00	I(4,2) = 0.1241E-02
V(1,3) = -0.7955E+00	I(1,3) = -0.3056E-01
V(2,3) = 0.5605E+00	I(2,3) = 0.2419E-01
V(3,3) = -0.8412E-01	I(3,3) = -0.6051E-02
V(4,3) = 0.2145E+00	I(4,3) = 0.7102E-02
V(1,4) = -0.9530E-01	I(1,4) = -0.1517E-02
V(2,4) = 0.1939E+00	I(2,4) = 0.9421E-02
V(3,4) = 0.1191E+00	I(3,4) = 0.8270E-02
V(4,4) = -0.9691E+00	I(4,4) = -0.2701E-01

VII. DISCUSSION

The numerical solution, implemented by the general purpose computer program of the report [9], has been found to be accurate for cases previously treated in the literature. Those cases previously considered were all for problems of some particular geometry, and all for single-mode transmission lines. Our solution is general in that it can treat multiconductor transmission lines of arbitrary cross sections in multilayered dielectric media. Our solution is also applicable to multiple dielectric media of other shapes, not necessarily layered, but our computer program is not written to handle such cases.

For the conductance matrix $[G]$, the solution is the same as for the previously treated loss-free solution, except that the real dielectric constants are replaced by the complex ones of (14). For the resistance matrix $[R]$, an extension of the usual perturbation method of [5] is used. This extension requires that first the modes of the multiconductor transmission line be determined from the eigenvalue equation (21), and second the attenuation constants of all modes be determined from (27). Then, if desired, the $[R]$ matrix for the multiconductor line is determined from (30). This solution uses the approximation of conductor surface resistance R_s for the metal surfaces.

REFERENCES

[1] C. Wei, R. F. Harrington, J. R. Mautz, and T. K. Sarkar, "Multiconductor transmission lines in multilayered dielectric media," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-32, p. 439, Apr. 1984.

[2] C. Wei and R. F. Harrington, "Extension of the multiconductor transmission line solution to zero-thickness conductors and to conductors between parallel ground planes," Electrical and Computer Engineering Department, Syracuse Univ., Syracuse, NY, Rep. TR-83-5, Mar. 1983.

[3] D. Kajfez, "Multiconductor transmission lines," Dep. of Electrical Engineering, University of Mississippi, University, MS, June 1972.

[4] S. Frankel, *Multiconductor Transmission Line Analysis.* Dedham, MA: Artech House, 1977.

[5] S. Ramo, J. R. Whinnery, T. Van Duzer, *Fields and Waves in Communication Electronics.* New York: Wiley, 1965.

[6] R. F. Harrington, *Time-Harmonic Electromagnetic Fields.* New York: McGraw-Hill, 1961.

[7] H. A. Wheeler, "Transmission-line properties of a strip on a dielectric sheet on a plane," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-25, pp. 631–647, Aug. 1977.

[8] R. A. Pucel, D. J. Masse, and C. P. Hartwig, "Losses in microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 342–350, June 1968.

[9] R. F. Harrington and C. Wei, "Losses on multiconductor transmission lines in multilayered dielectric media," Electrical and Computer Engineering Department, Syracuse Univ., Syracuse, NY, Rep. TR-83-16, Sept. 1983.

Roger F. Harrington (S'48–A'53–M'57–SM'62–F'68) was born in Buffalo, NY, on December 24, 1925. He received the B.E.E. and M.E.E. degrees from Syracuse University, Syracuse, NY, in 1948 and 1950, respectively, and the Ph.D. degree from Ohio State University, Columbus, OH, in 1952.

From 1945 to 1946, he served as an Instructor at the U.S. Naval Radio Materiel School, Dearborn, MI, and from 1948 to 1950 he was employed as an Instructor and Research Assistant at Syracuse University. While studying at Ohio State University, he served as a Research Fellow in the Antenna Laboratory. Since 1952 he has been on the faculty of Syracuse University, where he is presently Professor of Electrical Engineering. During 1959–1960 he was Visiting Associate Professor at the University of Illinois, Urbana, in 1964 he was Visiting Professor at the University of California, Berkeley, and in 1969 he was Guest Professor at the Technical University of Denmark, Lyngby, Denmark.

Dr. Harrington is a member of Tau Beta Pi, Sigma Xi, and the American Association of University Professors.

Cao Wei was born in Changsha, Hunan Province, China, in 1939. He graduated and received the B.S. degree from the Beijing Institute of Posts and Telecommunications in 1959.

After graduation, he taught mathematics for three years in the Nanjing Institute of Posts and Telecommunications, Nanjing, China. He then returned to the Beijing Institute of Posts and Telecommunication to study in the Department of Radio Telecommunication, from which he graduated in 1965. Since then, he has taught and done research in the areas of electromagnetics, microwave techniques, and antennas at the Nanjing Institute of Posts and Telecommunications. In September 1981, he came to the United States as a Visiting Scholar to undertake research with Professor R. F. Harrington in the Department of Electrical and Computer Engineering, Syracuse University.